Exercise 15

For the following exercises, consider this scenario: A town has an initial population of 75,000. It grows at a constant rate of 2,500 per year for 5 years.

If the function P is graphed, find and interpret the x- and y-intercepts.

Solution

Because the town's population grows at a constant rate, a linear function can be used to model it. The slope is 2500, the rate that the town's population increases per year, and the initial population is $75\,000$.

$$P(t) = 2500t + 75\,000$$

The y-intercept is the point at which the line crosses the y-axis. Set $t = 0$ to find where it does exactly.

$$P(0) = 2500(0) + 75\,000 = 75\,000$$

Therefore, the y-intercept is $(0, 75\,000)$. The x-intercept is the point at which the line crosses the x-axis. Set $P = 0$ and solve the equation for t.

$$0 = 2500t + 75\,000$$

$$-2500t = 75\,000$$

$$t = -\frac{75\,000}{2500} = -30$$

Therefore, the x-intercept is $(-30, 0)$. This means the population is zero 30 years prior to when the town's population is 75,000, assuming the constant rate of 2500 always holds.